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CONSTRAINTS ON PRIMORDIAL BLACK HOLES AND PRIMEVAL DENSITY PERTURBATIONS FROM THE EPOCH OF REIONIZATION

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ABSTRACT

We investigate the constraint on the abundance of primordial black holes (PBHs) and the spectral index n of primeval density perturbations given by the ionizing photon background at the epoch of reionization. Within the standard inflationary cosmogony, we show that the spectral index n of the power-law power spectrum of primeval density perturbations should be n <1.27. Since the universe is still optical thick at the reionization redshift $z \sim 6$ - 8, this constraint is independent of the unknown parameter of reheating temperature of the inflation. The ionizing photon background from the PBHs can be well approximated by a power law spectrum $J(\nu) \propto \nu^3$, which is greatly different from those given by models of massive stars and quasars.

Subject headings: cosmology: theory - large-scale structure of universe

1. INTRODUCTION

Primordial black holes (PBHs) form from the gravitational collapse when an initially dense region on super-horizon scales enters the horizon during the early radiation-dominated era (Carr & Hawking 1974, Carr 1975). In the inflationary universe, the abundance of PBHs is extremely sensitive to the spectral index n, which specifies the dependence of the power spectrum of primordial density fluctuations on the comoving wavenumber k, $|\delta_k|^2 \propto k^n$. Thus, a constraint on the abundance of PBHs will also yield a constraint on the index n.

The simplest constraint on the abundance of PBHs is given by the condition $\Omega_{bh} \leq 1$, i.e., the mass density of the PBHs should be less than the critical density of the universe. This condition results in $n < 1.5$ regardless of whether a black hole can evaporate completely, or ends with a Planck relic (Kim & Lee 1996). More effective constraints can be derived by comparing the particle emission from PBH evaporation with observed background. For instance, the cosmic background of γ-ray spectrum on ~ 100 Mev places a tightest constraint on the PBH density parameter $\Omega_{bh} \leq 10^{-8}$ (MacGibbon & Carr 1991), which yields the index $n < 1.25$ (Kim, Lee & MacGibbon 1999). However, these constraints depend on an additional parameter, the reheating temperature T_{rh}, or equivalently, the minimum mass of PBHs M_{min}, which is not well determined.

Recently, the epoch of the onset of the cosmic reionization is explored with absorption spectrum of high redshift ($5.7 < z < 6.3$) SDSS quasars (Fan et al. 2001, and references therein). The existence of the complete Lyα Gunn-Peterson trough in the absorption spectra of high redshift quasars indicates that the dark age probably ended at the redshift $z \simeq 6 - 8$. The ionizing photon background in the IGM at $z \sim 6$ is found to be more than 20 times lower than that at $z \sim 3$. These results motivate us to explore the constraints on the abundance of PBHs, and then on the primordial density perturbations with the low energy photon background at the epoch of reionization. We will show that this constraint is basically independent of the uncertain parameters T_{rh} or M_{min}. PBHs have been considered as a possible source of the cosmic reionization (Gibilisco 1996, 1997), but in these works, the mass function of PBHs is treated as free parameters. It cannot be used to constrain n.

This *Letter* is organized as follows. In §2, we briefly present the methods of calculating the mass function of the PBHs and the ionizing photon background from PBH's evaporation. The numerical results of the constraints on n from the observed high redshift ionizing background are given in §3. §4 are discussions and conclusions.

2. METHODS

2.1. *Mass function of PBHs*

For Gaussian primeval density perturbations, the mass function $n_{bh}(M_{bh})$ of PBHs at redshift z can be calculated by the same way as calculating the mass function of collapsed massive halos with the Press-Schechter formalism (Press & Schechter 1974). Accordingly, the number density of regions with density

contrast from δ to $\delta+d\delta$ and mass M to $M+dM$ at the time t_i of the onset of radiation-dominated epoch is

$$n(M,\delta)dMd\delta = \tag{1}$$
$$\sqrt{\frac{2}{\pi}}\frac{\rho_i}{M^2}\frac{1}{\sigma_R}\left|\frac{\partial \ln\sigma_R}{\partial \ln M}\right|\left|\frac{\delta^2}{\sigma_R^2}-1\right|e^{-\frac{\delta^2}{2\sigma_R^2}}dMd\delta,$$

where ρ_i is the cosmic mean mass density at t_i, M the mass within R, and σ_R the variance of the probability distribution function of initial Gaussian mass field smoothed by scale R. Since the mass function is normalized at ρ_i, i.e. $\int\int Mn(M,\delta)dMd\delta = \rho_i$, eq.(1) gives the physical number density of region (M,δ) at t_i. The comoving number density is $n(M,\delta)_{com} = n(M,\delta)(a_i/a_0)^3$, where $a(t)$ is the cosmic factor.

It has been shown (Carr 1975) that when a region (M,δ) enters the horizon at t_h, i.e. when M is about the same as horizon mass at t_h, this region will collapse to a black hole if the initial density contrast δ satisfies

$$\gamma(M/M_i)^{-2/3} \le \delta \le (M/M_i)^{-2/3}, \tag{2}$$

where parameter $\gamma \simeq 1/3$ and M_i is the horizon mass at t_i, i.e. the horizon mass at the beginning of the radiation-dominated. The mass of a black hole formed from this region is equal to $M_{bh} = \gamma^{1/2}M_i^{1/3}M^{2/3}$. Thus, the mass function of PBHs is

$$n_{bh}(M_{bh}) = \frac{dM}{dM_{bh}}\int_{\gamma(M/M_i)^{-2/3}}^{(M/M_i)^{-2/3}} n(M,\delta)d\delta = \tag{3}$$
$$\frac{3}{2}\sqrt{\frac{2}{\pi}}\frac{\gamma^{3/4}\rho_i M_i^{1/2}}{m_{bh}^{5/2}}\int_\gamma^1\left|\frac{\partial\ln\sigma_R}{\partial\ln M}\right|\left|\frac{\delta_h^2}{\sigma_h^2}-1\right|e^{-\frac{\delta_h^2}{2\sigma_h^2}}\frac{1}{\sigma_h}d\delta_h,$$

where $\delta_h = \delta(M/M_i)^{2/3}$ is the density contrast when the region (M,δ) crosses the horizon at t_h, and variance $\sigma_h = \sigma_R(M/M_i)^{2/3}$. Since $M \ge M_i$, the minimum mass of the PBH is $\gamma^{1/2}M_i$.

For a power law initial density perturbation with spectral index n, i.e. $\sigma_h \propto M^{(1-n)/6}$, we have $\partial\ln\sigma_R/\partial\ln M = (n+3)/6$, and eq.(3) becomes (Kim & Lee 1996)

$$n_{bh}(M_{bh}) = \tag{4}$$
$$\frac{n+3}{4}\left(\frac{2}{\pi}\right)^{1/2}\gamma^{7/4}\rho_i M_i^{1/2}M_{bh}^{-5/2}\sigma_h^{-1}e^{-\gamma^2/2\sigma_h^2}.$$

As usual, the variance σ_h will be normalized with the COBE observation (e.g., Fang & Xu 1999, Bugaev & Konishchev 2001).

In eq.(4), only the factors $\rho_i = 3/32\pi G t_i^2$ and $M_i = (4\pi/3)(ct_i)^3\rho_i$ are dependent on t_i. Thus, $n_{bh}(M_{bh}) \propto t_i^{-3/2}$. On the other hand, $(a_i/a_0)^3 \simeq (1+z_{eq})^3(t_i/t_{eq})^{3/2}$, where t_{eq} and z_{eq} are, respectively, the time and the redshift at the end of the radiation-dominated era. Thus, the comoving mass function $(a_i/a_0)^3 n_{bh}(M_{bh})$ is independent of t_i.

2.2. *Ionizing photon background from PBH's evaporation*

For a Schwarzschild black hole m, the Hawking emission rate for particles with spin s, energy from E to $E+dE$ and per degree of particle freedom is

$$\frac{dN}{dt} = \frac{1}{2\pi\hbar}\frac{\Gamma_s dE}{e^{E/k_BT(m)}-(-1)^{2s}}, \tag{5}$$

where $T(m) = \hbar c^3/8\pi G k_B m$ is the temperature of black hole m, and the dimensionless parameter Γ_s is the absorption probability of spin s particle by a black hole m (MacGibbon & Webber 1990; Page 1975). For low energy photon emission, i.e., $h\nu \ll k_b T(m)$, $\Gamma = 128G^4m^4\nu^4/3c^{12}$, and eq.(5) gives

$$\frac{d^2N}{d\nu dt} = f(\nu,m) = \frac{128G^4m^4\nu^4}{3c^{12}[e^{h\nu/k_BT(m)}-1]}. \tag{6}$$

The evaporation leads to the decreasing of black hole mass with the rate

$$\frac{dm}{dt} = -\frac{\alpha(m)}{m^2} \tag{7}$$

where $\alpha(m)$ takes into account all degrees of freedom of evaporated particles (MacGibbon 1991). Since $\alpha(m)$ is not a strong function of m, the time dependence of m for a black hole with initial mass M_{bh} at t_h is approximated by

$$m(M_{bh},t) \simeq [M_{bh}^3 - 3\alpha(m)t]^{1/3}, \tag{8}$$

if $t \gg t_h$.

Thus, the background photon energy flux $J(\nu, z_{obs})$ per unit frequency interval and per solid angle at redshift z_{obs} is as follows:

$$\frac{J(\nu,z_{obs})}{h\nu} = \frac{c}{4\pi}\int_{t_{eq}}^{t(z_{obs})} dt\left(\frac{a_{obs}}{a}\right)\left(\frac{a_i}{a_{obs}}\right)^3 \tag{9}$$
$$\int_{M_{min}}^{M_{max}} dM_{bh}n_{bh}(M_{bh})\exp[-\tau_{eff}(\nu,z_{obs},z)]$$
$$f[\nu(1+z)/(1+z_{obs}), m(M_{bh},t)],$$

M_{min} is given by $\max[\gamma^{1/2}M_i, m(t)]$, where $m(t)$ is the solution of $m(t) = \{3\alpha[m(t)]t\}^{1/3}$. The integral upper limit M_{max} in eq.(9) in not important, as the mass function $n_{bh}(M_{bh})$ is rapidly decreasing with the increasing of M_{bh}. $\tau_{eff}(\nu,z_{obs},z)$ is the opacity for photons emitted at z and observed at z_{obs} at the frequency ν.

3. RESULTS

Using eq.(9), we calculate the ionizing photon background at redshift $z_{obs} \sim 6$ - 8. The opacity τ_{eff} for photons observed at the hydrogen Lyman edge ($h\nu_0$ =13.6 ev) is dominated by HI photoelectric absorption. We will use the τ_{eff} estimated by the so-called high absorption model (Meiksin & Madau, 1993). It is

$$\begin{aligned} \tau(\nu_0, z_{obs}, z) \simeq & \quad (10) \\ & 0.0097(1+z_{obs})^{1.56}[(1+z)^{1.84}-(1+z_{obs})^{1.84}] \\ & -0.0068(1+z_{obs})^{1.56}[(1+z)^{0.4}-(1+z_{obs})^{0.4}] \\ & 8.06 \times 10^{-5}[(1+z)^{3.4}-(1+z_{obs})^{3.4}]. \end{aligned}$$

The optical depth given by eq.(10) is very short. For $z_{obs} = 6$, the mean transmission $e^{-\tau_{eff}}$ is less than 10^{-4} at $z \simeq 6.5$. That is, the effective flux $J(\nu_0)$ observed at $z = 6$ is only contributed by PBHs localized within the spatial range from $z = 6$ to 6.5. For $z = 8$, this range is about $z = 8$ to 8.25. Actually, eq.(10) is found from a fitting with observations at $z \sim 3$ - 4. However, there is no direct observation of the hydrogen Lyman edge absorption at redshifts $z > 6$. Nevertheless, eq.(10) is consistent with the Lyα transmission $e^{-\tau} = 0.004 \pm 0.003$ in the redshift range $[5.95 - 6.15]$ (Becker et al. 2001). Hence, it would be reasonable to estimate the optical depth at $z = 6$ - 6.5 by eq.(10).

The latest observation of the complete Gunn-Peterson troughs in the absorption spectrum of a z=6.28 quasar indicates that the photoionization rate at $z \sim 6$ is lower than 8.0×10^{-14}s^{-1} for Lyα, or 2.0×10^{-14}s^{-1} for Lyγ (Fan et al. 2001). Assuming the ionizing photons are from massive stars $J(\nu) \propto \nu^{-5}$ (Barkana & Loeb 2001), one can then find that the upper limits to the ionizing photon background $J_{-21} \equiv J(\nu)/10^{-21}$ ($J(\nu)$ in unit of erg cm^{-2} sec^{-1}Hz^{-1}sr^{-1}) at photon energy 13.6ev are, respectively, 5.3×10^{-2} and 1.3×10^{-2} for Lyα, and Lyγ, and at 24ev, are 3.1×10^{-3}, 7.8×10^{-4} for Lyα, and Lyγ, respectively.

Figure 1 presents the PBH contributed ionizing photon background J_{-21} at $z = 6$ as a function of the index n of the primordial fluctuation power spectrum. The upper limits to the ionizing photon background yield $n < 1.273$ for E=13.6ev, and $n < 1.270$ for E=24 ev.

It should be pointed out that these constraint basically are independent of t_i. Eq.(9) shows that $J(\nu_0)$ depends on t_i only via M_{min}, the truncation of PBH mass function at small mass side. On the other hand, the optical depth at $z = 6$ is very small. Only PBHs, which can survive to $z \simeq 6$, has contribution to $J(\nu_0)$. Thus, $J(\nu_0)$ is independent of M_{min}, if $M_{min} \leq 10^{12}$ g, as those PBHs have already evaporated at $z \simeq 6$ [eqs.(7) and (8)]. In inflationary universe, $t_i = 0.301 g_*^{-1/2} M_{pl}/T_{rh}^2$ (Kolb & Turner 1990). Thus, the M_{min}-independence leads to also T_{rh}-independence.

The M_{min} or T_{rh}-independence of $J(\nu_0)$ can be also seen from Fig. 2, which plots $dJ(\nu_0, z_{obs})/dm$ vs. m. It shows that $J(\nu_0)$ at $z = 6$ is mainly contributed by the evaporation of PBHs with mass $m \simeq 2 \times 10^{14}$ g. Therefore, $J(\nu_0)$ is independent of M_{min} if it is less than 2×10^{14} g. The peak 2×10^{14} g. is also consistent with the assumption of $h\nu \ll k_b T(m)$ used in eq.(6).

We calculate the flux $J(\nu, z)$ at $z = 6$ in the frequency range from $\nu = 3\times10^{15}$ to 6×10^{15}, which corresponds to photon energy range from 13.6 to 24.6 ev (HeI ionization). In this frequency interval one can still approximately use the optical depth τ_{eff} of eq.(10). The result is shown in Fig.3. The flux $J(\nu)$ can be well approximated by a power law spectrum $J(\nu) \propto \nu^3$. This spectrum is very different from the models of massive stars ($J(\nu) \propto \nu^{-5}$) and quasars ($J(\nu) \propto \nu^{-1.8}$, see Madau, Haardt, & Rees 1999), for both of which, the flux decreases with the increasing of ν. This is because $J(\nu)$ are mainly from the low energy tail of the photon emission by 10^{14}g black holes, whose temperature $T(m) \gg h\nu_0/k_b$. In the low energy tail, the intensity is increasing with ν.

In all the above calculations, the photons produced from the fragmentation of the PBH-emitted quarks and gluons are not considered (MacGibbon 1991). Since $J(\nu_0)$ is given by local PBHs, we need only to consider the fragmentation of quarks and gluons into low energy (~ 10 - 20 ev) photons. This problem is still quite unclear and uncertain. Since the fragmentation effect will lead to an increase of $J(\nu_0)$. Therefore, all above-mentioned upper limits on n will be held.

4. DISCUSSIONS AND CONCLUSIONS

In the standard inflationary model with a power-law power spectrum of the primeval density perturbations, we calculated the ionizing background flux of the PBHs. With the latest observed ionizing photon background flux at the redshift of reionization, we find that the spectral index of the primeval density perturbations should be $n < 1.27$, assuming that the background photon flux are emitted by massive stars. The difference is negligible if the background flux coming from quasars. If the parameter γ is taken to be 0.6 (Niemeyer & Jedamzik 1998), we have n <1.29. We also calculate $J(\nu)$ at $z = 8$. The result is almost the same as Fig. 3. However, there is no available data on the $J(\nu_0)$ at this redshift. If the reionization is really onset at $z \simeq 6$ - 8, $J(\nu_0)$ at $z = 8$ would be significantly less than that at $z = 6$,

one may have more tight constraint. All these constraints are independent of the reheating temperature T_{rh}, if $T_{rh} > 10^{10}$ Gev. The abundance of black hole given by this constraint is $\Omega_{bh} < 10^{-4}$, which is much stronger than the critical density constraint $\Omega_{bh} < 1$.

The current observations of the angular power spectrum of the temperature fluctuations of cosmic microwave background radiation (CMB) are found to be able to fit with a $n = 1$ power spectrum on angular scales from tens of degrees to sub-degrees. The PBH's constraint $n < 1.27$ shows that primeval density perturbations might still be scale invariant on scales much less than the direct observation of the CMB. Moreover, in the CMB data fitting, the parameters of the spectral index n and the optical depth of the CMB photons to the last scattering surface τ_c are degenerated (Stompor et al. 2001). That is, even when the fitted result is $n_{fitting} \simeq 1$, the original index may be $n > 1$ if $\tau_c > 0$. On the other hand, the constraint on n given by the PBH is independent of the optical depth τ_c.

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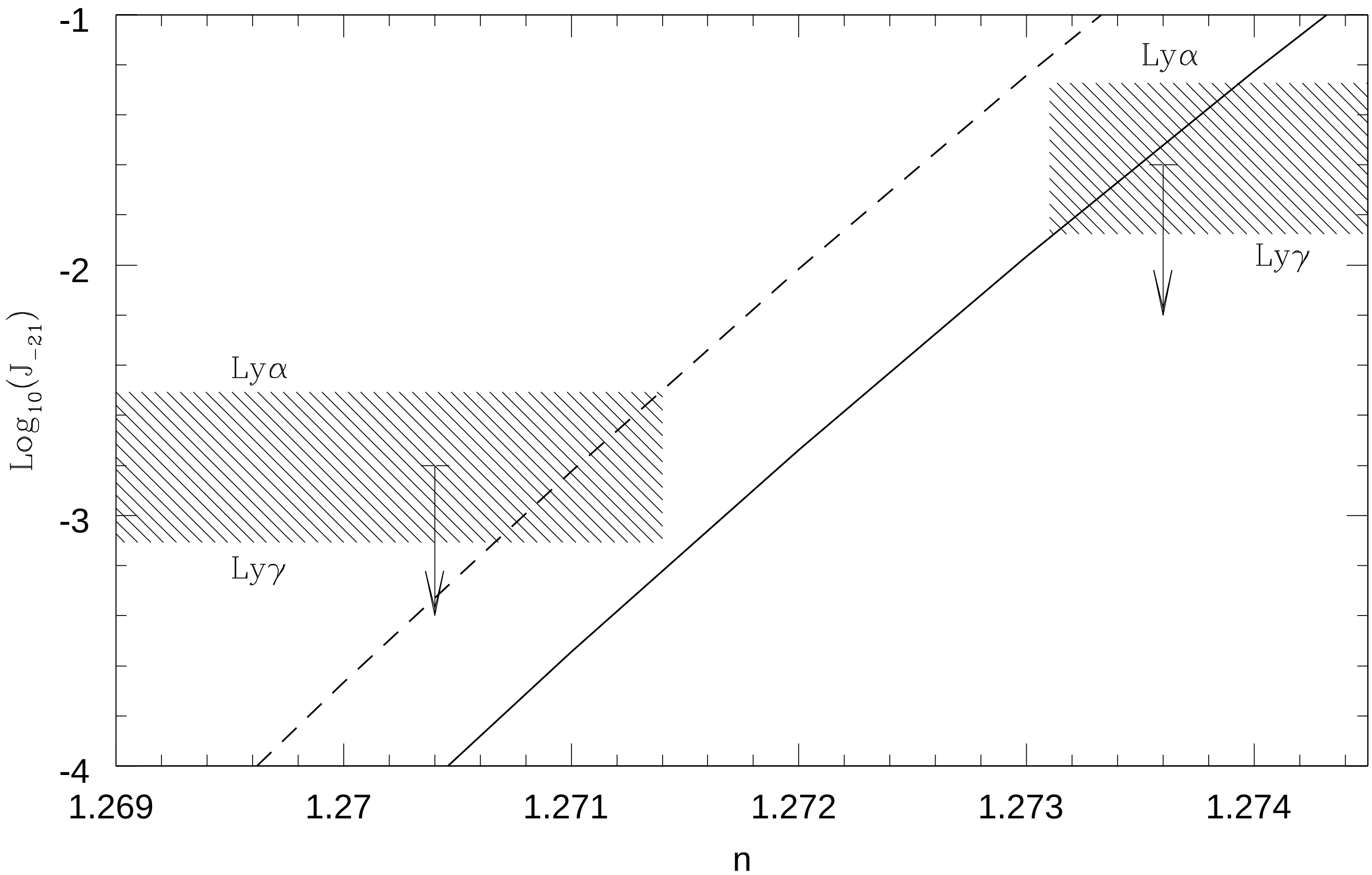
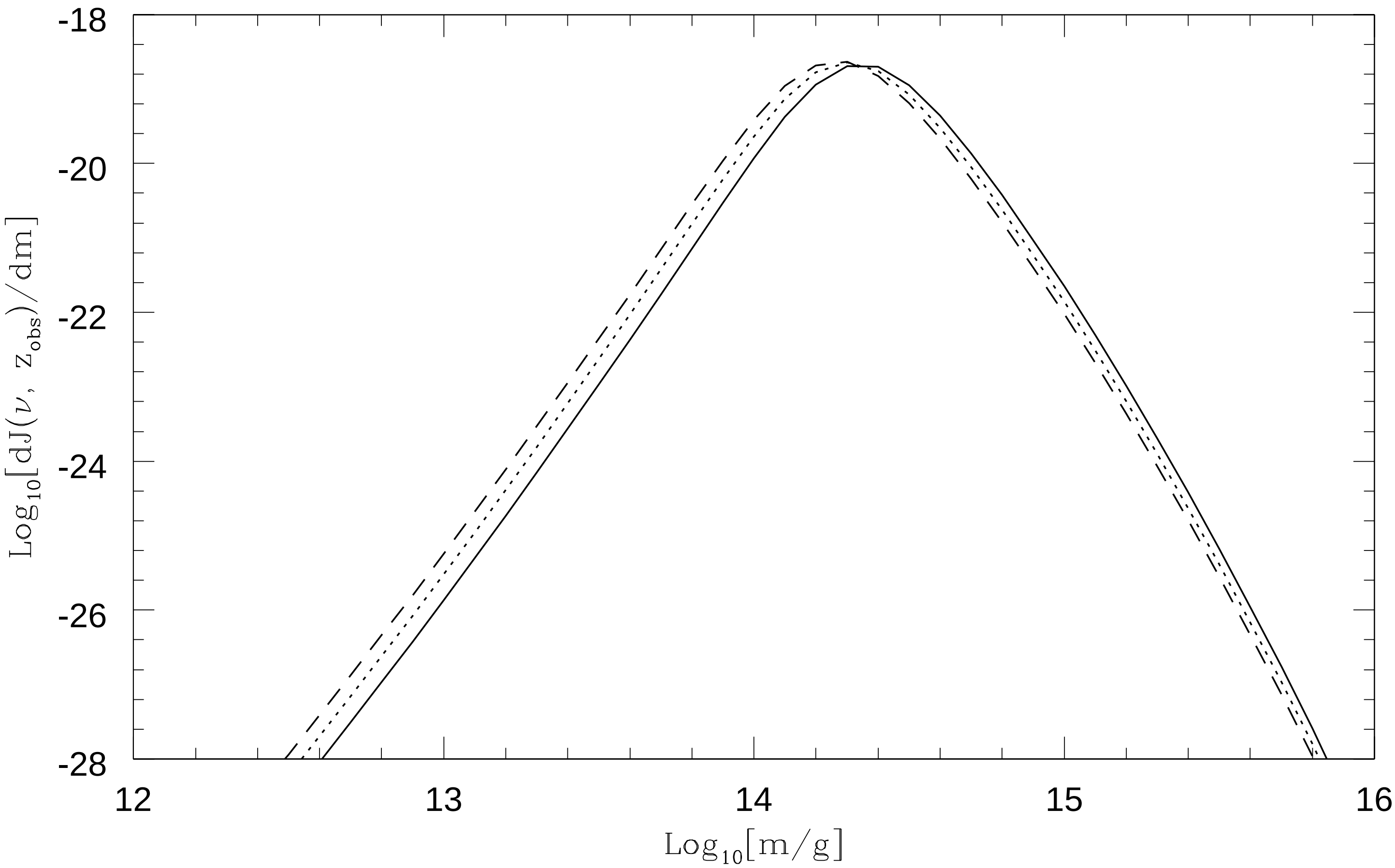
Fig. 1.— The PBH's ionizing background flux J_{-21} as a function of spectral index n of the primeval density perturbations at z=6: 1. photon energy E=13.6 ev (solid); 2. 24 ev (dashed). The observed upper limits to J_{-21} at E=13.6 and 24 ev are shown, respectively, by the shaded areas at top-right and bottom left. The upper and lower boundaries of the two areas refer to the limits given by Lyα and Lyγ absorption troughs. These limits are estimated with the model of massive stars.

Fig. 2.— The differential ionizing background flux $dJ(\nu_0, m)/dm$ vs. m, with the power spectrum index n=1.27. The solid, dotted, and dashed lines refer to cases of z=6, 8, and 10, respectively.

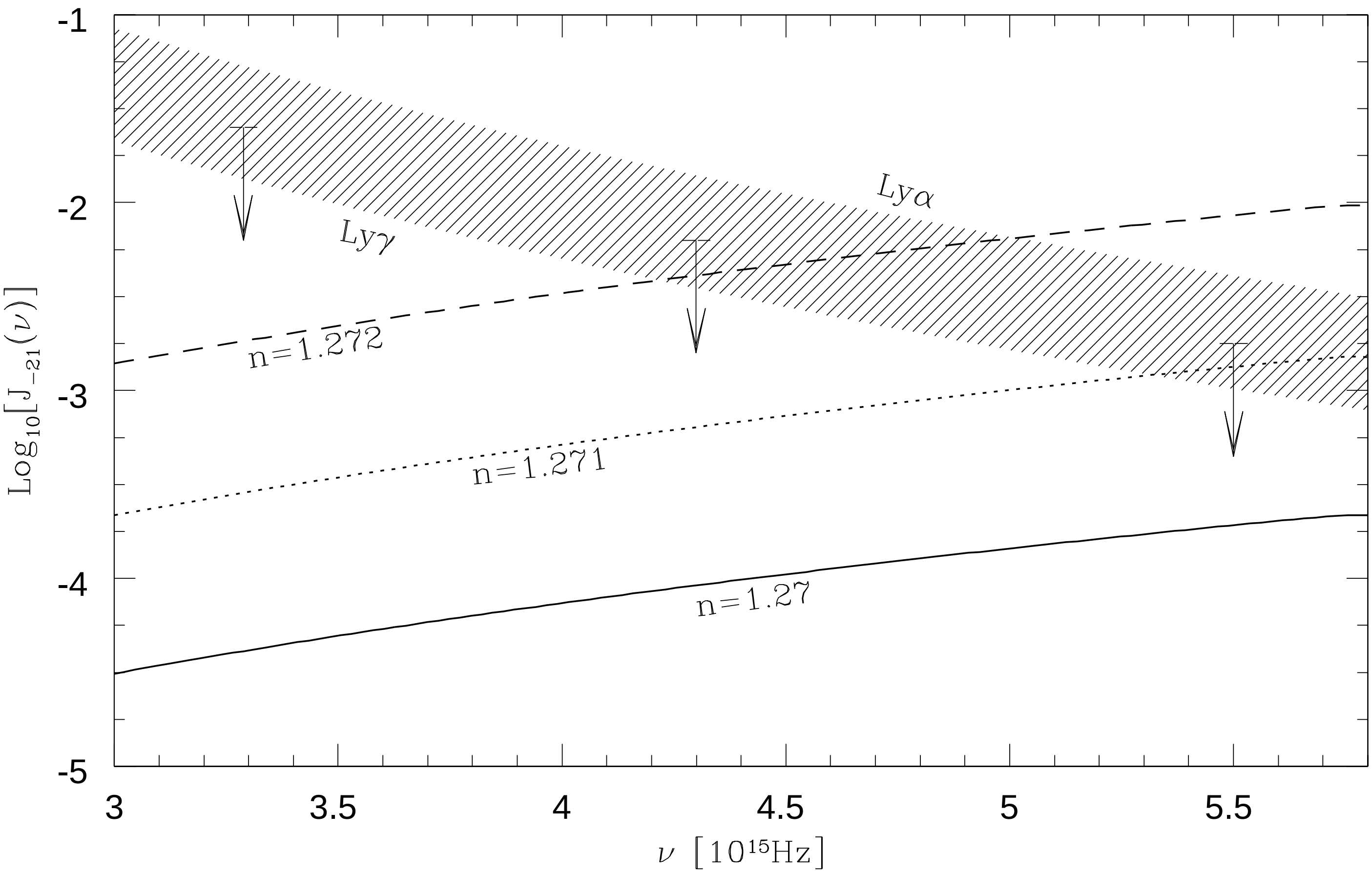
Fig. 3.— The spectrum of ionizing photon produced by PBHs. The shaded area is upper limits to the ionizing photon spectrum given by Lyα, and Lyγ in the massive star model.



Lyα
Lyγ
Lyα
Lyγ
-1
-2
-3
-4
1.269
1.27
1.271
1.272
1.273
1.274
n
$\mathrm{Log}_{10}(J_{-21})$



-18
-20
-22
-24
-26
-28
12
13
14
15
16
$\mathrm{Log_{10}[dJ(\nu,\ z_{obs})/dm]}$
$\mathrm{Log_{10}[m/g]}$



-1
-2
-3
-4
-5
3
3.5
4
4.5
5
5.5
$\mathrm{Log}_{10}[J_{-21}(\nu)]$
ν [10^{15}Hz]
Lyα
Lyγ
n=1.272
n=1.271
n=1.27